Hadronic Weak Decays of Heavy Mesons and Nonfactorization

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Abstract

The parameters $\chi_{1,2}$, which measure nonfactorizable soft gluon contributions to hadronic weak decays of mesons, are updated by extracting them from the data of $D, B \to PP, VP$ decays (P: pseudoscalar meson, V: vector meson). It is found that χ_2 ranges from -0.36 to -0.60 in the decays from $D \to \bar{K}\pi$ to $D^+ \to \phi \pi^+$, $D \to \bar{K}^*\pi$, while it is of order 10% with a positive sign in $B \to \psi K$, $D\pi$, $D^*\pi$, $D\rho$ decays. Therefore, the effective parameter a_2 is process dependent in charm decay, whereas it stays fairly stable in B decay. This implies the picture that nonfactorizable effects become stronger when the decay particles become less energetic after hadronization. As for $D, B \to VV$ decays, the presence of nonfactorizable terms in general prevents a possible definition of effective a_1 and a_2 . This is reenforced by the observation of a large longitudinal polarization fraction in $B \to \psi K^*$ decay, implying S-wave dominated nonfactorizable effects. The nonfactorizable term dominated by the S-wave is also essential for understanding the decay rate of $B^- \to D^{*0} \rho^-$. It is found that all nonfactorizable effects $A_1^{nf}/A_1^{BK^*}$, $A_1^{nf}/A_1^{B\rho}$, $A_1^{nf}/A_1^{BD^*}$ (nf standing for nonfactorization) are positive and of order 10%, in accordance with $\chi_2(B \to D(D^*)\pi(\rho))$ and $\chi_2(B \to \psi K)$. However, we show that in $D \to \bar{K}^* \rho$ decay nonfactorizable effects cannot be dominated by the Swave. A polarization measurement in the color- and Cabibbo-suppressed decay mode $D^+ \to \phi \rho^+$ is strongly urged in order to test if A_2^{nf}/A_2 plays a more pivotal role than A_1^{nf}/A_1 in charm decay.

1. Introduction

It is customary to assume that two-body nonleptonic weak decays of heavy mesons are dominated by factorizable contributions. Under this assumption, the spectator meson decay amplitude is the product of the universal parameter a_1 (for external W-emission) or a_2 (for internal W-emission), which is channel independent in D or B decays, and hadronic matrix elements which can be factorized as the product of two independent hadronic currents. The universal parameters a_1 and a_2 are related to the Wilson coefficient functions c_1 and c_2 by

$$a_1 = c_1 + \frac{1}{N_c}c_2, \qquad a_2 = c_2 + \frac{1}{N_c}c_1,$$
 (1)

with N_c being the number of colors. It is known that the bulk of exclusive nonleptonic charm decay data cannot be explained by this factorization approach [1]. For example, the predicted ratio of the color-suppressed mode $D^0 \to \bar{K}^0\pi^0$ and color-favored decay $D^0 \to K^-\pi^+$ is in violent disagreement with experiment. This signals the importance of the nonfactorizable effects.

The leading nonfactorizable contribution arises from the soft gluon exchange between two color-octet currents

$$O_c = \frac{1}{2} (\bar{q}_1 \lambda^a q_2) (\bar{q}_3 \lambda^a q_4), \tag{2}$$

where $(\bar{q}_1\lambda^a q_2)$ stands for $\bar{q}_1\gamma_\mu(1-\gamma_5)\lambda^a q_2$. For $M\to PP$, VP decays (P): pseudoscalar meson, V: vector meson), the nonfactorizable effect amounts to a redefinition of the parameters a_1 and a_2 [2], a_2

$$a_1 \to c_1 + c_2(\frac{1}{N_c} + \chi_1), \quad a_2 \to c_2 + c_1(\frac{1}{N_c} + \chi_2),$$
 (3)

where χ_1 and χ_2 denote the contributions of O_c to color-favored and color-suppressed decay amplitudes respectively relative to the factorizable ones. For example, for $D_s^+ \to \phi \pi^+$, $D^+ \to \phi \pi^+$ decays,

$$\chi_1(D_s^+ \to \phi \pi^+) = \frac{\langle \phi \pi^+ | \frac{1}{2} (\bar{u} \lambda^a d) (\bar{s} \lambda^a c) | D_s^+ \rangle}{\langle \phi \pi^+ | (\bar{u} d) (\bar{s} c) | D_s^+ \rangle_f},$$

$$\chi_2(D^+ \to \phi \pi^+) = \frac{\langle \phi \pi^+ | \frac{1}{2} (\bar{u} \lambda^a c) (\bar{s} \lambda^a s) | D^+ \rangle}{\langle \phi \pi^+ | (\bar{u} c) (\bar{s} s) | D^+ \rangle_f}.$$
(4)

The subscript f in Eq.(4) denotes a factorizable contribution:

$$\langle \phi \pi^{+} | (\bar{u}d)(\bar{s}c) | D_{s}^{+} \rangle_{f} = 2m_{\phi} f_{\pi}(\varepsilon^{*} \cdot p_{D_{s}}) A_{0}^{D_{s}\phi}(m_{\pi}^{2}),$$

$$\langle \phi \pi^{+} | (\bar{u}c)(\bar{s}s) | D^{+} \rangle_{f} = m_{\phi} f_{\phi}(\varepsilon^{*} \cdot p_{D}) F_{1}^{D\pi}(m_{\phi}^{2}),$$
(5)

¹Note that our definition of χ_1 and χ_2 is different from r_1 and r_2 defined in [3] by a factor of 2.

where ε_{μ} is the polarization vector of the ϕ meson, and we have followed Ref.[4] for the definition of form factors. The nonfactorizable contributions have the expressions

$$\langle \phi \pi^{+} | \frac{1}{2} (\bar{u} \lambda^{a} d) (\bar{s} \lambda^{a} c) | D_{s}^{+} \rangle = 2 m_{\phi} f_{\pi} (\varepsilon^{*} \cdot p_{D_{s}}) A_{0}^{nf} (m_{\pi}^{2}),$$

$$\langle \phi \pi^{+} | \frac{1}{2} (\bar{u} \lambda^{a} c) (\bar{s} \lambda^{a} s) | D^{+} \rangle = m_{\phi} f_{\phi} (\varepsilon^{*} \cdot p_{D}) F_{1}^{nf} (m_{\phi}^{2}), \tag{6}$$

with the superscript nf referring to nonfactorizable contributions. It is clear that

$$\chi_1(D_s^+ \to \phi \pi^+) = \frac{A_0^{nf}(m_\pi^2)}{A_0^{D_s \phi}(m_\pi^2)}, \quad \chi_2(D^+ \to \phi \pi^+) = \frac{F_1^{nf}(m_\phi^2)}{F_1^{D\pi}(m_\phi^2)}.$$
 (7)

That is, χ simply measures the fraction of nonfactorizable contributions to the form factor under consideration.

Although we do not know how to calculate χ_1 and χ_2 from first principles, we do anticipate that [3]

$$|\chi(B \to PP)| < |\chi(D \to PP)| < |\chi(D \to VP)|, \tag{8}$$

based on the reason that nonperturbative soft gluon effects become more important when the final-state particles move slower, allowing more time for significant final-state interactions after hadronization. As a consequence, it is obvious that $a_{1,2}$ are in general not universal and that the rule of discarding $1/N_c$ terms [5], which works empirically well in $D \to \bar{K}\pi$ decay, cannot be safely extrapolated to $B \to D\pi$ decay as $|\chi(B \to D\pi)|$ is expected to be much smaller than $|\chi(D \to \bar{K}\pi) \sim -\frac{1}{3}|$ (the c.m. momentum in $D \to \bar{K}\pi$ being 861 MeV, to be compared with 2307 MeV in $B \to D\pi$) and hence a large cancellation between $1/N_c$ and $\chi(B \to D\pi)$ is not expected to happen. The recent CLEO observation [6] that the rule of discarding $1/N_c$ terms is not operative in $B \to D(D^*)\pi(\rho)$ decays is therefore not stunning. Only the fact that $\chi(B \to D\pi)$ is positive turns out to be striking.

Unlike the PP or VP case, it is not pertinent to define $\chi_{1,2}$ for $M \to VV$ decay as its general amplitude consists of three independent Lorentz scalars:

$$A[M(p) \to V_1(\varepsilon_1, p_1)V_2(\varepsilon_2, p_2)] \propto \varepsilon_{\mu}^*(\lambda_1)\varepsilon_{\nu}^*(\lambda_2)(\hat{A}_1 g^{\mu\nu} + \hat{A}_2 p^{\mu} p^{\nu} + i\hat{V}\epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta}), \tag{9}$$

where \hat{A}_1 , \hat{A}_2 , \hat{V} are related to the form factors A_1 , A_2 and V respectively. Since a priori there is no reason to expect that nonfactorizable terms weight in the same way to S-, P- and D-waves, namely $A_1^{nf}/A_1 = A_2^{nf}/A_2 = V^{nf}/V$, we thus cannot define χ_1 and χ_2 . Consequently, it is in general not possible to define an effective a_1 or a_2 for $M \to VV$ decays once nonfactorizable effects are taken into account [7]. In the factorization approach, the fraction of polarization, say Γ_L/Γ (L: longitudinal polarization) in $B \to \psi K^*$ decay, is independent

of the parameter a_2 . As a result, if an effective a_2 can be defined for $B \to \psi K^*$, it will lead to the conclusion that nonfactorizable terms cannot affect the factorization prediction of Γ_L/Γ at all. It was realized recently that all the known models in the literature in conjunction with the factorization hypothesis fail to reproduce the data of Γ_L/Γ or the production ratio $\Gamma(B \to \psi K^*)/\Gamma(B \to \psi K)$ or both [8,9]. Evidently, if we wish to utilize nonfactorizable effects to resolve the puzzle with Γ_L/Γ , a key ingredient will be the nonexistence of an effective a_2 for $B \to \psi K^*$.

In short, there are two places where the factorization hypothesis can be unambiguiously tested: (i) To extract the parameters a_1 and a_2 from the experimental measurements of $M \to PP$, VP to see if they are process independent. (ii) To measure the fraction of longitudinal polarization in $M \to VV$ decay and compare with the factorization prediction. Any failure of them will indicate a breakdown of factorization.

The purpose of the present paper is threefold. (i) The parameters χ_1 and χ_2 have been extracted in Ref.[3] (see also [10]). Here we wish to update the values of $\chi_{1,2}$ using the q^2 dependence of form factors suggested by QCD-sum-rule calculations and other theoretical arguments. (ii) It was recently advocated by Kamal and Sandra [7] that the assumption that in $B \to \psi K^*$ decay the nonfactorizable amplitude contributes only to S-wave final states, namely $A_1^{nf} \neq 0$, $A_2^{nf} = V^{nf} = 0$, will lead to a satisfactory explanation of the data of $\Gamma(B \to \psi K^*)/\Gamma(B \to \psi K)$ and Γ_L/Γ . We would like to show that this very assumption is also essential for understanding the ratio $\mathcal{B}(B^- \to D^{*0}\rho^-)/\mathcal{B}(\bar{B}^0 \to D^{*+}\rho^-)$, which cannot be explained satisfactorily in previous work. (iii) Contrary to the B meson case, we will demonstrate that the assumption of S-wave dominated nonfactorizable terms does not work in $D \to VV$ decay.

2. Nonfactorizable contributions in $D, B \rightarrow PP, VP$ decays

Because of the presence of final-state interactions (FSI) and the nonspectator contributions (W-exchange and W-annihilation), it is generally not possible to extract the nonfactorization parameters $\chi_{1,2}$ except for a very few channels. Though color-suppressed decays, for example, $D^0 \to \bar{K}^0(\bar{K}^{*0})\pi^0(\rho^0)$ are conventionally classified as Class II modes [11], color-flavored decay $D^0 \to K^-\pi^+$ will bring some important contribution to $D^0 \to \bar{K}^0\pi^0$ via FSI. This together with the small but not negligible W-exchange amplitude renders the determination of a_2 from $D^0 \to \bar{K}^0\pi^0$ impossible. Therefore, in order to determine a_1 and especially a_2 we should focus on the exotic channels e.g. $D^+ \to \bar{K}^0\pi^+$, $\pi^+\pi^0$, and the decay modes with one single isospin component, e.g. $D^+ \to \pi^+\phi$, $D_s^+ \to \pi^+\phi$, where nonspectator contributions are absent and FSI are presumably negligible.

We next write down the relations between $\chi_{1,2}$ and form factors

$$\chi_{1}(D \to \bar{K}\pi) = \frac{F_{0}^{nf}(m_{\pi}^{2})}{F_{0}^{DK}(m_{\pi}^{2})}, \qquad \chi_{2}(D \to \bar{K}\pi) = \frac{F_{0}^{nf}(m_{K}^{2})}{F_{0}^{D\pi}(m_{K}^{2})},
\chi_{1}(D \to \bar{K}^{*}\pi) = \frac{A_{0}^{nf}(m_{\pi}^{2})}{A_{0}^{DK^{*}}(m_{\pi}^{2})}, \qquad \chi_{2}(D \to \bar{K}^{*}\pi) = \frac{F_{1}^{nf}(m_{K^{*}}^{2})}{F_{1}^{D\pi}(m_{K^{*}}^{2})},
\chi_{1}(D \to \bar{K}\rho) = \frac{F_{1}^{nf}(m_{\rho}^{2})}{F_{1}^{DK}(m_{\rho}^{2})}, \qquad \chi_{2}(D \to \bar{K}\rho) = \frac{A_{0}^{nf}(m_{K}^{2})}{A_{0}^{D\rho}(m_{K}^{2})},
\chi_{1}(D_{s}^{+} \to \phi\pi^{+}) = \frac{A_{0}^{nf}(m_{\pi}^{2})}{A_{0}^{Ds\phi}(m_{\pi}^{2})}, \qquad \chi_{2}(D^{+} \to \phi\pi^{+}) = \frac{F_{1}^{nf}(m_{\phi}^{2})}{F_{1}^{D\pi}(m_{\phi}^{2})}.$$
(10)

It is clear that only the three form factors F_0 , F_1 and A_0 entering into the decay amplitudes of $M \to PP$, VP. A consideration of the heavy quark limit behavior of the form factors suggests that the q^2 dependence of F_1 (A_2) is different from that of F_0 (A_0 and A_1) by an additional pole factor [12]. Indeed, QCD-sum-rule calculations have implied a monopole behavior for $F_1(q^2)$ [13-16] and an approximately constant F_0 [15]. With a dipole form factor A_2 , as shown by a recent QCD-sum-rule analysis [16], we will thus assume a monopole behavior for A_0 .

Unlike the decays $D^+ \to \pi^+ \phi$, $D_s^+ \to \pi^+ \phi$ which are described by a single quark diagram, we cannot extract $\chi_{1,2}$ from the data of $D^+ \to \bar K^0 \pi^+$, $\bar K^0 \rho^+$, $\bar K^{*0} \pi^+$ alone without providing further information. For example, the decay amplitude of $D^+ \to \bar K^0 \pi^+$ reads

$$A(D^{+} \to \bar{K}^{0}\pi^{+}) = \frac{G_{F}}{\sqrt{2}} V_{cs}^{*} V_{ud} [a_{1}(m_{D}^{2} - m_{K}^{2}) f_{\pi} F_{0}^{DK}(m_{\pi}^{2}) + a_{2}(m_{D}^{2} - m_{\pi}^{2}) f_{K} F_{0}^{D\pi}(m_{K}^{2})], (11)$$

which consists of external W-emission and internal W-emission amplitudes. We will therefore make a plausible assumption that $\chi_1 \sim \chi_2$ so that $\chi(D \to \bar{K}\pi)$ can be determined from the measured rate of $D^+ \to \bar{K}^0\pi^+$. Since the extraction procedure is already elucidated in Ref.[3], here we will simply present the results (only the central values being quoted) followed by several remarks

$$\chi_2(D \to \bar{K}\pi) \simeq -0.36,$$

$$\chi_2(D \to \bar{K}^*\pi) \simeq -0.61,$$

$$\chi_2(D^+ \to \phi\pi^+) \simeq -0.44,$$
(12)

where we have used the following quantities:

$$c_1(m_c) = 1.26,$$
 $c_2(m_c) = -0.51,$
 $f_{\pi} = 132 \,\text{MeV},$ $f_K = 160 \,\text{MeV},$ $f_{K^*} = 220 \,\text{MeV},$ $f_{\phi} = 237 \,\text{MeV},$
 $F_0^{DK}(0) = F_1^{DK}(0) = 0.77 \pm 0.04 \,[17],$ $F_0^{D\pi}(0) = F_1^{D\pi}(0) = 0.83 \,[18],$ (13)
 $A_1^{DK^*}(0) = 0.61 \pm 0.05,$ $A_2^{DK^*}(0) = 0.45 \pm 0.09 \,[17],$ $\Rightarrow A_0^{DK^*}(0) = 0.70,$

and the Particle Data Group [19] for the decay rates of various decay modes.

Several remarks are in order. (i) As pointed out by Soares [10], the solutions for χ are not uniquely determined. For example, the other possible solution for $\chi_2(D \to \bar{K}\pi)$ is -1.18. To remove the ambiguities, we have assumed that nonfactorizable corrections are small compared to the factorizable ones. (ii) Assuming $A_0^{D\rho}(0) = A_0^{DK^*}(0)$, we find from the decay $D^+ \to \bar{K}^0 \rho^+$ that $\chi(D \to \bar{K}\rho) \approx -1.5$, wich is unreasonably too large. We do not know how to resolve this problem except for noting that thus far there is only one measurement of this decay mode [20]. (iii) To determine $\chi_1(D_s^+ \to \phi \pi^+)$ requires a better knowledge of the form factor $A_0^{D_s\phi}$ and the branching ratio of $D_s^+ \to \phi \pi^+$. Unfortunatly, a direct measurement of them is still not available. Assuming $A_0^{D_s\phi}(0) \approx A_0^{DK^*}(0)$ and $\mathcal{B}(D_s^+ \to \phi \pi^+) = (3.5 \pm 0.4)\%$ [19], we get $\chi_1(D_s^+ \to \phi \pi^+) \approx -0.60$. So in general nonfactorizable terms are process or class dependent, and satisfy the relation $|\chi(D \to PP)| < |\chi(D \to VP)|$ as expected. (iv) Since $\chi_2(D \to \bar{K}\pi)$ is close to $-\frac{1}{3}$, it is evident that a large cancellation between $1/N_c$ and $\chi_2(D \to \bar{K}\pi)$ occurs. This is the dynamic reason why the large- N_c approach operates well for $D \to \bar{K}\pi$ decay. However, this is no longer the case for $D \to VP$ decays. The predicted branching ratios in $1/N_c$ expansion are

$$\mathcal{B}(D^+ \to \bar{K}^{*0}\pi^+) = 0.3\%, \qquad \mathcal{B}(D^+ \to \bar{K}^0\rho^+) = 16\%,$$

$$\mathcal{B}(D^+ \to \bar{K}^{*0}\rho^+) = 17\%, \qquad \mathcal{B}(D^+ \to \phi\pi^+) = 0.4\%, \tag{14}$$

to be compared with data [19]

$$\mathcal{B}(D^+ \to \bar{K}^{*0}\pi^+)_{\text{expt}} = (2.2 \pm 0.4)\%, \qquad \mathcal{B}(D^+ \to \bar{K}^0\rho^+)_{\text{expt}} = (6.6 \pm 2.5)\%,$$

$$\mathcal{B}(D^+ \to \bar{K}^{*0}\rho^+)_{\text{expt}} = (4.8 \pm 1.8)\%, \qquad \mathcal{B}(D^+ \to \phi\pi^+)_{\text{expt}} = (0.67 \pm 0.08)\%. \quad (15)$$

Consider the decay $D^+ \to \bar{K}^{*0}\pi^+$ as an example. Its amplitude is given by

$$A(D^+ \to \bar{K}^{*0}\pi^+) = \sqrt{2}G_F V_{cs}^* V_{ud}[a_1 f_\pi m_{K^*} A_0^{DK^*}(m_\pi^2) + a_2 f_{K^*} m_{K^*} F_1^{D\pi}(m_{K^*}^2)].$$
 (16)

Since the interference is destructive and $f_{K^*}F_1^{D\pi} > f_\pi A_0^{DK^*}$, a large $|a_2|$ is needed in order to enhance the branching ratio of $D^+ \to \bar{K}^{*0}\pi^+$ from 0.3% to 2.2%. (Note that a_1 is relatively insensitive to the nonfactorizable effects.) This in turn implies a negative $(\frac{1}{N_c} + \chi_2)$ and hence $\chi_2(D \to \bar{K}^*\pi) < -\frac{1}{3}$. Therefore, we are led to conclude that the leading $1/N_c$ expansion cannot be a universal approach for the nonleptonic weak decays of the meson. However, the fact that substantial nonfactorizable effects which contribute destructively with the subleading $1/N_c$ factorizable contributions are required to accommodate the data of charm decay means that, as far as charm decays are concerned, the large- N_c approach greatly improves the naive factorization method in which $\chi_{1,2}=0$; the former approach amounts to having a universal nonfactorizable term $\chi_{1,2}=-1/N_c$.

We next turn to $B \to D(D^*)\pi(\rho)$ decays. Though both nonspectator and FSI effects are known to be important in charm decays, it is generally believed that they do not play a significant role in bottom decays as the decay particles are moving fast, not allowing adequate time for FSI. This gives the enormous advantage that it is conceivable to determine a_1 and a_2 separately from $B \to D(D^*)\pi(\rho)$ decays. Using the heavy-flavor-symmetry approach for heavy-light form factors and assuming a monopole extrapolation for F_1 , F_2 , and F_3 , F_4 , and an approximately constant F_4 , as suggested by QCD-sum-rule calculations and some theoretical arguments [21], we found from the CLEO data that [21] F_4

$$a_1(B \to D^{(*)}\pi(\rho)) = 1.01 \pm 0.06,$$

 $a_2(B \to D^{(*)}\pi(\rho)) = 0.23 \pm 0.06.$ (17)

Taking $c_1(m_b) = 1.11$ and $c_2(m_b) = -0.26$ leads to

$$\chi_1(B \to D^{(*)}\pi(\rho)) \simeq 0.05, \quad \chi_2(B \to D^{(*)}\pi(\rho)) \simeq 0.11.$$
 (18)

Since $(\frac{1}{N_c} + \chi_{1,2}) = (a_{1,2} - c_{1,2})/c_{2,1}$ and $|c_2| << |c_1|$, it is clear that the determination of χ_1 is far more uncertain than χ_2 : it is very sensitive to the values of a_1 , c_1 and c_2 . We see from (18) that nonfactorizable effects become less important in B decays, as what expected [see (8)]. However, a positive $\chi_2(B \to D(D^*)\pi(\rho))$, which is necessary to explain the constructive interference in $B^- \to D^0(D^{*0})\pi^-(\rho^-)$ decays, appears to be rather striking. A recent light cone QCD-sum-rule calculation [22] following the framework outlined in [23] fails to reproduce a positive $\chi_2(B \to D\pi)$. This tantalizing issue should be resolved in the near future.

For $B \to \psi K$ decays, we found [21]

$$\left| a_2(B^- \to \psi K^-) \right| = 0.235 \pm 0.018, \quad \left| a_2(B^0 \to \psi K^0) \right| = 0.192 \pm 0.032.$$
 (19)

The combined value is

$$a_2(B \to \psi K) = 0.225 \pm 0.016,$$
 (20)

where its sign should be positive, as we have argued in [21]. (It was advocated by Soares [10] that an analysis of the contribution of $B \to \psi K$ to the decay $B \to K \ell^+ \ell^-$ can be used to remove the sign ambiguity of a_2 .) It follows that

$$\chi_2(B \to \psi K) = \frac{F_1^{nf}(m_\psi^2)}{F_1^{BK}(m_\psi^2)} \simeq 0.10,$$
(21)

²Contrary to the charmed meson case, the variation of $a_{1,2}$ from $B \to D\pi$ to $D^*\pi$ and $D\rho$ decays is negligible (see Table IV of [21]).

which is in accordance with $\chi_2(B \to D^{(*)}\pi(\rho))$.

Finally, it is very interesting to note that, in contrast to charm decays, the large- N_c approach is even worse than the naive factorization method in describing $B \to D(D^*)\pi(\rho)$ decays as $\chi_2(B \to D^{(*)}\pi(\rho))$ is small but positive.

3. Nonfactorizable contributions in $B \to \psi K^*$, $D^*\rho$ decays

As stressed in the Introduction, in general one cannot define $\chi_{1,2}$ and hence an effective $a_{1,2}$ for $M \to VV$ decays unless the nonfactorizable terms weight in the same manner in all three partial waves. It was pointed out recently that there are two experimental data, namely the production ratio $R \equiv \Gamma(B \to \psi K^*)/\Gamma(B \to \psi K)$ and the fraction of longitudinal polarization Γ_L/Γ in $B \to \psi K^*$, which cannot be accounted for simultaneously by all commonly used models within the framework of factorization [8,9]. The experimental results are

$$R = 1.74 \pm 0.39 \ [6], \quad \frac{\Gamma_L}{\Gamma} = 0.78 \pm 0.07,$$
 (22)

where the latter is the combined average of the three measurements:

$$\left(\frac{\Gamma_L}{\Gamma}\right)_{B \to \psi K^*} = \begin{cases}
0.97 \pm 0.16 \pm 0.15, & \text{ARGUS [24];} \\
0.80 \pm 0.08 \pm 0.05, & \text{CLEO [6];} \\
0.66 \pm 0.10^{+0.08}_{-0.10}, & \text{CDF [25].}
\end{cases}$$
(23)

Irrespective of the production ratio R, all the existing models fail to produce a large longitudinal polarization fraction [8,9]. This strongly implies that the puzzle with Γ_L/Γ can only be resolved by appealing to nonfactorizable effects. ³ However, if the relation $A_1^{nf}/A_1 = A_2^{nf}/A_2 = V^{nf}/V$ holds, then an effective a_2 can be defined for $B \to \psi K^*$ and the prediction of Γ_L/Γ will be the same as that in the factorization approach as the polarization fraction is independent of a_2 . Consequently, nonfactorizable terms should contribute differently to S-, P- and D-wave amplitudes if we wish to explain the observed Γ_L/Γ .

The large longitudinal polarization fraction observed by ARGUS and CLEO suggests that the decay $B \to \psi K^*$ is almost all S-wave. To see this, we write down the $B \to \psi K^*$

³An interesting observation was made recently in [26] that the factorization assumption in $B \to \psi K(K^*)$ is not ruled out and the data can be accommodated by the heavy-flavor-symmetry approach for heavy-light form factors provided that the $A_1(q^2)$ form factor is frankly decreasing. To our knowledge, a decreasing A_1 with q^2 is ruled out by several recent QCD-sum-rule analyses (see e.g. [16]). Using the same approach for heavy-light form factors but the q^2 dependence of form factors given in [21], we found that R = 1.84 and $\Gamma_L/\Gamma = 0.56$ [21]. Evidently, the factorization approach is still difficult to explain the observed large polarization fraction.

amplitude

$$A[B(p) \to \psi(p_1)K^*(p_2)] = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left(c_2 + \frac{c_1}{3} \right) f_{\psi} m_{\psi} \varepsilon_{\mu}^* (\psi) \varepsilon_{\nu}^* (K^*) [\hat{A}_1 g^{\mu\nu} + \hat{A}_2 p^{\mu} p^{\nu} + i \hat{V} \epsilon^{\mu\nu\alpha\beta} p_{1\alpha} p_{2\beta}],$$
(24)

with

$$\hat{A}_{1} = (m_{B} + m_{K^{*}}) A_{1}^{BK^{*}} (m_{\psi}^{2}) \left[1 + \kappa \frac{A_{1}^{nf}(m_{\psi}^{2})}{A_{1}^{BK^{*}}(m_{\psi}^{2})} \right],$$

$$\hat{A}_{2} = -\frac{2}{(m_{B} + m_{K^{*}})} A_{2}^{BK^{*}} (m_{\psi}^{2}) \left[1 + \kappa \frac{A_{2}^{nf}(m_{\psi}^{2})}{A_{2}^{BK^{*}}(m_{\psi}^{2})} \right],$$

$$\hat{V} = -\frac{2}{(m_{B} + m_{K^{*}})} V^{BK^{*}} (m_{\psi}^{2}) \left[1 + \kappa \frac{V^{nf}(m_{\psi}^{2})}{V^{BK^{*}}(m_{\psi}^{2})} \right],$$
(25)

and $\kappa = c_1/(c_2 + \frac{1}{3}c_1)$. It is easily seen that we will have an effective $a_2 = c_2 + c_1(\frac{1}{3} + \chi_2)$ if the nonfactorizable terms happen to satisfy the relation $A_1^{nf}/A_1 = A_2^{nf}/A_2 = V^{nf}/V = \chi_2$. The decay rate of this mode is of the form

$$\Gamma(B \to \psi K^*) \propto (a - b\tilde{x})^2 + 2(1 + c^2\tilde{y}^2),$$
 (26)

where

$$a = \frac{m_B^2 - m_{\psi}^2 - m_{K^*}^2}{2m_{\psi}m_{K^*}}, \qquad b = \frac{2m_B^2 p_c^2}{m_{\psi}m_{K^*}(m_B + m_{K^*})^2}, \quad c = \frac{2m_B p_c}{(m_B + m_{K^*})^2},$$

$$\tilde{x} = \frac{A_2^{BK^*}(m_{\psi}^2) + \kappa A_2^{nf}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2) + \kappa A_1^{nf}(m_{\psi}^2)}, \qquad \tilde{y} = \frac{V^{BK^*}(m_{\psi}^2) + \kappa V^{nf}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2) + \kappa A_1^{nf}(m_{\psi}^2)}, \tag{27}$$

with p_c being the c.m. momentum. The longitudinal polarization fraction is then given by

$$\frac{\Gamma_L}{\Gamma} = \frac{(a - b\tilde{x})^2}{(a - b\tilde{x})^2 + 2(1 + c^2\tilde{y}^2)}.$$
(28)

If the decay is an almost S-wave, one will have $\Gamma_L/\Gamma \sim a^2/(a^2+2)=0.83$. Since $\kappa >> 1$, \tilde{x} (D-wave) and \tilde{y} (P-wave) can be suppressed by assuming that, as first postulated in [7], in $B \to \psi K^*$ decay the nonfactorizable amplitude contributes only to S-wave final states; that

is, ⁴

$$A_1^{nf} \neq 0, \quad A_2^{nf} = V^{nf} = 0.$$
 (29)

The rational for this assumption is given in [7].

With the assumption (29), the branching ratio followed from (24) is

$$\mathcal{B}(B \to \psi K^*) = 0.0288 \left| \left(c_2 + \frac{c_1}{3} \right) A_1^{BK^*} (m_{\psi}^2) \right|^2 \left[(a\xi - bx)^2 + 2(\xi^2 + c^2 y^2) \right]$$
 (30)

with

$$x = \frac{A_2^{BK^*}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2)}, \quad y = \frac{V^{BK^*}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2)}, \quad \xi = 1 + \kappa \frac{A_1^{nf}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2)}, \tag{31}$$

where uses of $|V_{cb}| = 0.040$ and $\tau(B) = 1.52 \times 10^{-12} s$ have been made. It follows that

$$\frac{\Gamma_L}{\Gamma} = \frac{(a\xi - bx)^2}{(a\xi - bx)^2 + 2(1 + c^2y^2)}.$$
(32)

We use the measured branching ratio $\mathcal{B}(B \to \psi K^*) = (0.172 \pm 0.030)\%$ [6] to determine the ratio $A_1^{nf}(m_{\psi}^2)/A_1^{BK^*}(m_{\psi}^2)$, which is found to be

$$\frac{A_1^{nf}(m_{\psi}^2)}{A_1^{BK^*}(m_{\psi}^2)} \simeq 0.08\,, (33)$$

which we have used $A_1^{BK^*}(m_{\psi}^2) = 0.41$, $A_2^{BK^*}(m_{\psi}^2) = 0.36$, $V^{BK^*}(m_{\psi}^2) = 0.72$ [21] and discarded the other possible solution $A_1^{nf}/A_1^{BK^*} = -0.22$ for its "wrong" sign, recalling that F_1^{nf}/F_1^{BK} is positive [cf. Eq.(21)]. The predicted longitudinal polarization fraction is $\Gamma_L/\Gamma = 0.73$, which is in accordance with experiment.

The assumption of negligible nonfactorizable contributions to P- and D-waves also turns out to be essential for understanding the decay rate of $B^- \to D^{*0}\rho^-$ or the ratio $R_4 \equiv \mathcal{B}(B^- \to D^{*0}\rho^-)/\mathcal{B}(\bar{B}^0 \to D^{*+}\rho^-)$. The issue arises as follows. In Ref.[21] we have determined a_1 and a_2 from $B \to D\pi$, $D^*\pi$, $D\rho$ decays and obtained a consistent ratio a_2/a_1 from

$$\frac{\Gamma_L}{\Gamma} = \frac{(a - bx)^2}{(a - bx)^2 + 2(1 + c^2\bar{y}^2)},$$

with $\bar{y} = (V^{BK^*}(m_{\psi}^2) + \kappa V^{nf}(m_{\psi}^2))/A_1^{BK^*}(m_{\psi}^2)$ and x being defined in (31). It is clear that in order to get a large longitudinal polarization fraction one needs a negative V^{nf}/V ! Using the numerical values a = 3.164, b = 1.304, x = 0.89, we find $(\Gamma_L/\Gamma)_{\rm max} = 0.67$. The prediction $\Gamma_L/\Gamma = 0.65$ given by [27] is one standard deviation from experiment (22).

⁴ A different approach for nonfactorizable effects adopted in Ref.[27] amounts to $A_1^{nf}=A_2^{nf}=0$ and $V^{nf}\neq 0$. It follows from Eq.(28) that

channel to channel: 0.24 ± 0.10 , 0.24 ± 0.14 , 0.21 ± 0.08 (see Table IV of [21]). Assuming factorization, we got $a_2/a_1 = 0.34 \pm 0.13$ from $B \to D^* \rho$ decay, which deviates somewhat from above values. In the presence of S-wave dominated nonfactorizable contributions, it is no longer possible to define an effective a_1 and a_2 for $B \to D^* \rho$ decay. Therefore, the quantities to be compared with are A_1^{nf}/A_1 in $B \to D^* \rho$ decay and χ_2 in $B \to D\pi$, $D^*\pi$, $D\rho$. A straightforward calculation yields (see [21] for the factorizable case)

$$R_4 = \frac{\tau(B^-)}{\tau(B^0)} \left(1 + 2\eta \frac{H_1}{H} + \eta^2 \frac{H_2}{H} \right), \tag{34}$$

with

$$H = (\hat{a}\hat{\xi} - \hat{b}\hat{x})^{2} + 2(\hat{\xi}^{2} + \hat{c}^{2}\hat{y}^{2}),$$

$$H_{1} = (\hat{a}\hat{\xi} - \hat{b}\hat{x})(\hat{a}\hat{\xi}' - \hat{b}'\hat{x}') + 2(\hat{\xi}\hat{\xi}' + \hat{c}\hat{c}'\hat{y}\hat{y}'),$$

$$H_{2} = (\hat{a}\hat{\xi}' - \hat{b}'\hat{x}')^{2} + 2(\hat{\xi}'^{2} + \hat{c}'^{2}\hat{y}'^{2}),$$

$$\eta = \frac{m_{D^{*}}(m_{B} + m_{\rho})}{m_{\rho}(m_{B} + m_{D^{*}})} \frac{f_{D^{*}}}{f_{\rho}} \frac{A_{1}^{B\rho}(m_{D^{*}}^{2})}{A_{1}^{BD^{*}}(m_{\rho}^{2})} \frac{c_{2} + \frac{1}{3}c_{1}}{c_{1} + \frac{1}{3}c_{2}},$$

$$\hat{\xi} = 1 + \frac{c_{2}}{c_{1} + \frac{1}{3}c_{2}} \frac{A_{1}^{nf}(m_{\rho}^{2})}{A_{1}^{BD^{*}}(m_{\rho}^{2})},$$

$$\hat{\xi}' = 1 + \frac{c_{1}}{c_{2} + \frac{1}{3}c_{1}} \frac{A_{1}^{nf}(m_{D^{*}}^{2})}{A_{1}^{B\rho}(m_{D^{*}}^{2})},$$

$$(35)$$

where \hat{a} , \hat{b} , \hat{c} are obtained from a, b, c respectively in (27), \hat{x} , \hat{y} from x, y in (31) by replacing $\psi \to D^*$, $K^* \to \rho$, and \hat{b}' , \hat{c}' , \hat{x}' , \hat{y}' are obtained from \hat{b} , \hat{c} , \hat{x} , \hat{y} respectively by replacing $D^* \leftrightarrow \rho$; for instance $\hat{x}' = A_2^{B\rho}(m_{D^*}^2)/A_1^{B\rho}(m_{D^*}^2)$. Assuming $A_1^{nf}/A_1^{BD^*} \sim A_1^{nf}/A_1^{B\rho}$ and fitting (34) to the experimental value $R_4 = (1.68 \pm 0.35)\%$ [6], we get

$$\frac{A_1^{nf}(m_{D^*}^2)}{A_1^{B\rho}(m_{D^*}^2)} \sim \frac{A_1^{nf}(m_{\rho}^2)}{A_1^{BD^*}(m_{\rho}^2)} \simeq 0.12.$$
 (36)

We see that the S-wave dominated nonfactorizable effect in $B \to \psi K^*$ and $B \to D^* \rho$ decays is of order 10%, consistent with $\chi_2(B \to \psi K)$ and $\chi_2(B \to D(D^*)\pi(\rho))$.

4. Nonfactorizable contributions in $D \to \bar{K}^* \rho$ decay

We have shown in the previous section that S-wave dominated nonfactorizable terms are needed to explain the large longitudinal polarization fraction observed in $B \to \psi K^*$ and the ratio $\mathcal{B}(B^- \to D^{*0}\rho^-)/\mathcal{B}(\bar{B}^0 \to D^{*+}\rho^-)$. However, we shall see in this section that the assumption (29) is no longer applicable to $D \to \bar{K}^*\rho$ decay. An experimental measurement of $D^+ \to \bar{K}^{*0}\rho^+$ and $D^0 \to \bar{K}^{*0}\rho^0$ by Mark III [28] shows that (i) the decay $D^+ \to \bar{K}^{*0}\rho^+$ is a mixture of longitudinal and transverse polarization consistent with a pure

S-wave amplitude, ⁵ and (ii) $D^0 \to \bar{K}^{*0} \rho^0$ is almost all transverse, requiring a cancellation between the longitudinal S-wave and D-wave.

We first consider the decay $D^+ \to \bar{K}^{*0} \rho^+$, whose amplitude is given by

$$A(D^{+}(p) \to \bar{K}^{*0}(p_{1})\rho^{+}(p_{2})) = \frac{G_{F}}{\sqrt{2}}V_{cs}^{*}V_{ud}\varepsilon_{\mu}^{*}(K^{*})\varepsilon_{\nu}^{*}(\rho)[\tilde{A}_{1}g^{\mu\nu} + \tilde{A}_{2}p^{\mu}p^{\nu} + i\tilde{V}\epsilon^{\mu\nu\alpha\beta}p_{1\alpha}p_{2\beta}], (38)$$

where

$$\tilde{A}_{1} = \left(c_{1} + \frac{c_{2}}{3}\right) f_{\rho} m_{\rho} (m_{D} + m_{K^{*}}) \left(1 + \frac{c_{2}}{c_{1} + \frac{1}{3}c_{2}} \frac{A_{1}^{nf}(m_{\rho}^{2})}{A_{1}^{DK^{*}}(m_{\rho}^{2})}\right) A_{1}^{DK^{*}}(m_{\rho}^{2})
+ \left(c_{2} + \frac{c_{1}}{3}\right) f_{K^{*}} m_{K^{*}} (m_{D} + m_{\rho}) \left(1 + \frac{c_{1}}{c_{2} + \frac{1}{3}c_{1}} \frac{A_{1}^{nf}(m_{K^{*}}^{2})}{A_{1}^{D\rho}(m_{K^{*}}^{2})}\right) A_{1}^{D\rho}(m_{K^{*}}^{2}), \quad (39)$$

and \tilde{A}_2 (\tilde{V}) is obtained from \tilde{A}_1 with the replacements $A_1 \to A_2$ ($A_1 \to V$), $(m_D + m_{K^*}) \to -2/(m_D + m_{K^*})$ and $(m_D + m_\rho) \to -2/(m_D + m_\rho)$. Since $A_1^{nf}/A_1^{DK^*}$ and $A_1^{nf}/A_1^{D\rho}$ are expected to be negative [see Eq.(12)], it is obvious that if nonfactorizable terms are dominated by the S-wave, it will imply a more severe destructive interference in the S-wave amplitude than in P- and D-wave amplitudes, in contradiction to the observation that this decay is almost all S-wave. The branching ratio is calculated to be

$$\mathcal{B}(D^+ \to \bar{K}^{*0}\rho^+) = 0.10 \left| \left(c_1 + \frac{1}{3}c_2 \right) A_1^{DK^*}(m_\rho^2) \right|^2 (H' + 2\eta' H_1' + \eta'^2 H_2'), \tag{40}$$

with the expressions of η' , H', $H'_{1,2}$ analogous to η , H, $H_{1,2}$ in (35). A fit of (40) to the Mark III data for the branching ratio (37) gives rise to (assuming $A_1^{nf}/A_1^{DK^*} \sim A_1^{nf}/A_1^{D\rho}$)

$$\frac{A_1^{nf}(m_\rho^2)}{A_1^{DK^*}(m_\rho^2)} \sim \frac{A_1^{nf}(m_{K^*}^2)}{A_1^{D\rho}(m_{K^*}^2)} \approx -0.98,$$
(41)

which is uncomfortably too large. ⁶ Moreover, the P-wave branching ratio is predicted to be 2.0×10^{-2} , in disagreement with experiment [28]

$$\mathcal{B}(D^+ \to \bar{K}^{*0}\rho^+)_{P-\text{wave}} < 0.5 \times 10^{-2}.$$
 (42)

It thus appears to us that an almost S-wave $D^+ \to \bar K^{*0} \rho^+$ implies that

$$\left| \frac{A_2^{nf}}{A_2^{DK^*(\rho)}} \right|, \quad \left| \frac{V^{nf}}{V^{DK^*(\rho)}} \right| \gtrsim \left| \frac{A_1^{nf}}{A_1^{DK^*(\rho)}} \right|. \tag{43}$$

$$\mathcal{B}(D^+ \to \bar{K}^{*0}\rho^+) = \begin{cases} (4.8 \pm 1.2 \pm 1.4)\%, & \text{Mark III [28];} \\ (2.3 \pm 1.2 \pm 0.9)\%, & \text{E691 [29].} \end{cases}$$
(37)

Recall that model calculations tend to give a very large branching ratio of 17% [see Eq.(14)].

 $^6{\rm A}$ fit to the E691 measurement (37) for the branching ratio yields an even larger value: $A_1^{nf}/A_1^{DK^*}\sim A_1^{nf}/A_1^{D\rho}\approx -1.41$.

 $^{^5}$ The other measurement by E691 [29] disagrees severely with Mark III on the branching ratio

Taking $A_1^{nf}/A_1 = A_2^{nf}/A_2 = V^{nf}/V = \chi(D \to \bar{K}^*\rho)$ as an illustration, we obtain

$$\chi(D \to \bar{K}^* \rho) \approx -0.65 \tag{44}$$

and $\mathcal{B}(D^+ \to \bar{K}^{*0}\rho^+)_{P-\text{wave}} = 2.0 \times 10^{-3}$, which are certainly more plausible than before.

Another indication for the failure of the S-wave dominated hypothesis for nonfactorizable effects comes from the decay $D^0 \to \bar{K}^{*0} \rho^0$, where \bar{K}^{*0} and ρ^0 are completely transversely polarized, implying a large D-wave which is compensated by the longitudinal S-wave. Recall that the factorizable $D \to VV$ amplitudes have the sailent feature:

$$|S$$
-wave amplitude $|S|$ -wave amplitude $|S|$ -wave amplitude $|S|$ (45)

Since the color-suppressed D-wave amplitude of $D^0 \to \bar{K}^{*0} \rho^0$ is proportional to $[1 + c_1/(c_2 + \frac{1}{3}c_1)(A_2^{nf}/A_2^{D\rho})]$, a large D-wave thus indicates a negative A_2^{nf}/A_2 and

$$\left| \frac{A_1^{nf}}{A_1} \right| << \left| \frac{A_2^{nf}}{A_2} \right|, \quad \text{or } \frac{A_1^{nf}}{A_1} \approx 0, \quad \frac{A_2^{nf}}{A_2} \neq 0.$$
 (46)

Therefore, we see that nonfactorizable terms in charm decay are consistently to be negative [cf. Eqs.(12) and (44)]. Unfortunately, at this point we cannot make a further quantitative analysis due to unknown final-state interactions and W-exchange contributions. A measurement of helicities in $D^0 \to \bar{K}^{*0}\rho^0$, $D^+ \to \phi\rho^+$ will be greatly helpful to pin down the issue. In particular, the color- and Cabibbo-suppressed mode $D^+ \to \phi\rho^+$ is very ideal for this purpose since it is not subject to FSI and nonspectator effects. A polarization measurement in this decay is thus strongly urged (though difficult) in order to test if A_2^{nf}/A_2 plays a more essential role than A_1^{nf}/A_1 in charm decay.

5. Discussion and conclusion

The factorization assumption for hadronic weak decays of mesons can be tested on two different grounds: (i) to extract the effective parameters a_1 and especially a_2 from $M \to PP$, VP decays to see if they are process independent, and (ii) to measure helicities in $M \to VV$ decay. Using the q^2 dependence of form factors suggested by QCD-sum-rule calculations and by some theoretical arguments, we have updated our previous work. It is found that a_2 is evidently not universal in charm decay. The parameter χ_2 , which measures the nonfactorizable soft-gluon effect on the color-suppressed deacy amplitude relative to the factorizable one, ranges from $-\frac{1}{3}$ to -0.60 in the decays from $D \to \bar{K}\pi$ to $D^+ \to \phi\pi^+$, $D \to \bar{K}^*\pi$. By contrast, the variation of a_2 in $B \to \psi K$, $B \to D(D^*)\pi(\rho)$ is negligible and nonfactorizable terms $\chi_2(B \to \psi K)$, $\chi_2(B \to D^{(*)}\pi(\rho))$ are of order 10% with a positive sign. The pattern for the relative magnitudes of nonfactorizable effects

$$|\chi(B\to PP,VP)|<|\chi(D\to PP)|<|\chi(D\to VP)|$$

is thus well established. This means that nonperturbative soft gluon effects become more important when the final states are less energetic, allowing more time for final-state interactions. This explains why a_2 is class (PP or VP mode) dependent in charm decay, whereas it stays fairly stable in B decay.

Taking factorization as a benchmark, we see that the nonfactorizable terms necessary for describing nonleptonic D and B decays are in opposite directions from the factorization framework. On the one hand, the leading $1/N_c$ expansion, which amounts to a universal $\chi = -\frac{1}{3}$, improves the naive factorization method for charm decays. On the other hand, the naive factorization hypothesis works better than the large- N_c assumption for B decays because nonfactorizable effects are small, being of order 10%. The fact that χ is positive makes it even more clear that the large- N_c approach cannot be extrapolated from D to B physics. Theoretically, the next important task for us is to understand why χ is negative in D decay, while it becomes positive in B decay.

As for $M \to VV$ decay, a priori effective $a_{1,2}$ cannot be defined since, as pointed out by Kamal and Sandra, its amplitude (factorizable and nonfactorizable) involves three independent Lorentz scalars, corresponding to S, P and D waves. This turns out to be a nice trade-off for solving the puzzle with the large longitudinal polarization fraction Γ_L/Γ observed in $B \to \psi K^*$, which cannot be accounted for by the factorization hypothesis or by nonfactorizable effects weighted in the same way in all three partial waves, namely $A_1^{nf}/A_1 = A_2^{nf}/A_2 = V^{nf}/V$. A large Γ_L/Γ can be achieved if $B \to \psi K^*$ is almost all Swave, implying that nonfactorizable contributions are dominated by the S-wave. The same assumption is also needed for understanding the ratio $\mathcal{B}(B^- \to D^{*0}\rho^-)/\mathcal{B}(\bar{B}^0 \to D^{*+}\rho^-)$. We found that all nonfactorizable terms $A_1^{nf}/A_1^{BK^*}$, $A_1^{nf}/A_1^{B\rho}$, $A_1^{nf}/A_1^{BD^*}$ are of order 10% consistent with $\chi_2(B \to D(D^*)\pi(\rho))$ and $\chi_2(B \to \psi K)$.

Surprisingly, the assumption of S-wave dominated nonfactorizable effects is not operative in $D \to \bar{K}^* \rho$ decay, which exhibits again another disparity between B and D physics. We found that A_2^{nf}/A_2 should play a more pivotal role than A_1^{nf}/A_1 in charm decay. We thus urge experimentalists to measure helicities in the color- and Cabibbo-suppressed decay mode $D^+ \to \phi \rho^+$ decay to gain insight in the nonfactorizable effects in $D \to VV$ decay.

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